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## MECHANICS.

When this issue was made up, solutions had been received for 285–8–9 and 292. Solutions of 266, 268, 269, 271 to 275, inclusive, and 277 to 279 inclusive are desired.

**293. Proposed by B. F. FINKEL, Drury College.**

A man of weight $W$ stands on smooth ice; prove that if, when he gradually parts his legs, kept straight, with his feet in contact with the ice, the pressure of his feet on the ice be constant, his head will descend with uniform acceleration; and that, if $f$ be the acceleration of his head, when his feet exert no pressure on the ice, their pressure on the ice, were $f'$ the acceleration of his head, would be equal to $\frac{f-f'}{f}W$.

(From WALTON's *Problems in Theoretical Mechanics.*)

**294. Proposed by EMMA GIBSON, Drury College.**

A sphere, revolving about a diameter and not acted on by any extraneous force, expands symmetrically; prove that its vis viva varies inversely as its moment of inertia about a diameter.

## NUMBER THEORY.

When this issue was made up, solutions had been received for 206–7, 210, 213. Solutions of 192 and 196 are desired.

**215. Proposed by R. D. CARMICHAEL, Indiana University.**

Find one or more values of $n$ such that a polygon of $n$ sides shall have the number of its diagonals equal to the cube of an integer.

**216. Proposed by ELIJAH SWIFT, Princeton, N. J.**

If $p$ is a prime $> 3$, show that $\Sigma_{a=1}^{a=p-1} 1/a \equiv 0 \pmod{p^2}$, where $1/a \equiv x$, if $ax \equiv 1 \pmod{p^2}$

**217. Proposed by E. T. BELL, Seattle, Wash.**

(i) If $r$ is a prime greater than 2, and $p \equiv 2^a r + 1$ is prime, the only solution, when $n$ is greater than 2, of $x^n - y^n = p$, is $n = 3$, $x = 2$, $y = 1$.

(ii) The only primes that are simultaneously of the forms $4k + 1$ and $3^m - 2^m$ are 1 and 5.

(iii) Generalize (ii).

# SOLUTIONS OF PROBLEMS.

## ALGEBRA.

**385. Proposed by J. F. LAWRENCE, Stillwater, Oklahoma.**

Show that if $p$ is prime and $> 3$, $(2p)! - 2 \cdot p!p!$ is divisible by $p^5$.

### SOLUTION BY ELIJAH SWIFT, Princeton, N. J.

Dividing the above expression by $2p \cdot p!$ we have to show that

$$(1) \qquad (2p-1)(2p-2)(2p-3)\cdots(2p-(p-1)) - (p-1)!$$

is divisible by $p^3$.

Consider the polynomial $P(x) = (x-1)(x-2)(x-3)\cdots(x-p+1)$.[1] If this is expanded in powers of $x$, we obtain an expression of the form

$$x^{p-1} - A_1x^{p-2} + A_2x^{p-3} + \cdots + A_{p-3}x^2 - A_{p-2}x + A_{p-1},$$

---

[1] See BACHMANN, *Niedere Zahlentheorie*, p. 155. I have changed signs, but evidently the same results hold.

where

$$
\begin{aligned}
A_1 &= 1 + 2 + 3 + \cdots + p - 1\\
A_2 &= 1\cdot 2 + 1\cdot 3 + \cdots + 2\cdot 3 + \cdots + (p-2)(p-1)\\
&\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\\
&\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\\
A_{p-1} &= 1\cdot 2\cdot 3\cdots(p-1).
\end{aligned}
$$

It is proved by Bachmann that each of these $A$'s, except $A_{p-1}$, is divisible by $p$.

Obviously, from the expression of $P(x)$ in factor form $P(p) = (p-1)!$ It follows that $P(p) - (p-1)! = p^{p-1} - A_1 p^{p-2} + \cdots + A_{p-3}p^2 - A_{p-2}p = 0$. If then $A_{p-3}$ is divisible by $p$, $A_{p-2}$ will be divisible by $p^2$. If $p = 3$, however, $A_{p-3} = 1$, or rather there is no $A_{p-3}$, and our reasoning breaks down.

Now expand (1). We shall evidently have $P(2p) - (p-1)! = (2p)^{p-1} - A_1(2p)^{p-2} + \cdots + A_{p-3}(2p)^2 - A_{p-2}(2p)$. But $A_{p-3}$ is divisible by $p$, and $A_{p-2}$ by $p^2$. Therefore, (1) is divisible by $p^3$, and $(2p)! - 2\cdot p!p!$ by $p^5$.

**398. Proposed by R. D. CARMICHAEL, Indiana University.**

In the equation $x^3 + \alpha x + \beta = 0$, $\alpha$ is an integer divisible by $p^2$ and $\beta$ is an integer divisible by $p$, $p$ being a prime number. Prove that $\beta$ is divisible by $p^3$ if the equation is reducible.

### SOLUTION BY ELMER SCHUYLER, Brooklyn, New York.

Let $(x^2 + lx + m)(x + n) \equiv x^3 + \alpha x + \beta$. Then since $\alpha = p^2t$ and $\beta = pr$, $l = -n$; $m - n^2 = p^2t$; $mn = pr$. Hence, $m = pr/n$ and $pr/n - n^2 = p^2t$, or $pr - n^3 = p^2nt$. Consequently, $n$ is divisible by $p$ and $n = ps$, since $n$ clearly is an integer. Whence $pr - p^3s^3 = p^3st$ or $r/p^2 = st - s^3$. Hence, we have $\beta = p^3(st - s^3)$.

Also solved by B. LIBBY, G. W. HARTWELL, A. M. HARDING, and ELIJAH SWIFT.

**399. Proposed by W. H. BUSSEY, University of Minnesota.**

$A$ borrows from $B$ \$1,500 and pays back \$34 a month for 63 months. If the last payment closes the account, what rate of interest has $A$ been paying.

### SOLUTION BY GEO. W. HARTWELL, Hamline University.

This problem is equivalent to the following: $B$ pays $A$ \$1,500 for an annuity of \$34 per month to run 63 months. What is the rate realized?

Let $r$ = the annual rate. Then $r/12$ = the monthly rate. Hence,

$$
34\left(1+\frac{r}{12}\right)^{62} + 34\left(1+\frac{r}{12}\right)^{61} + \cdots + 34 \equiv 34\left[\frac{\left(1+\frac{r}{12}\right)^{63} - 1}{\frac{r}{12}}\right] = 1{,}500,
$$

or the total number of dollars realized.

Solving this equation by the usual method, we find $r = 14.31$ per cent.

Also solved by G. Y. SASNOW, ALBERT R. NAUER, WILLIAM CULLUM, W. C. EELLS, and CLIFFORD N. MILLS.

**400. Proposed by C. N. SCHMALL, New York City.**

Sum the series $1 + 2x + 3x^2 + 4x^3 + \cdots$.

(BROMWICH, *Infinite Series*, p. 129, ex. 1.)